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THE FORCE ACTING ON A DROP IN A CONDUCTING LIQUID WHEN THERE IS AN ELECTRIC CURRENT†

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The motion of a spherical drop in a gradient unsteady flow of viscous liquid moving in the magnetic field of a spatially non-uniform electric current flowing through the liquid, which contains drops and which generates Lorentz vortex forces, is considered. The conductivity of the drop differs from that of the carrying liquid, in view of which each drop produces a local non-uniformity in the electromagnetic-field distribution, which has a considerable effect on the flow around the drop. A formula is obtained for the force acting on the drop using the method of solving Stokes equations containing vortex volume forces [1]. In existing publications (a bibliography is given in [2]) the effect of a uniform electric current on the hydrodynamic streamline pattern and the drag when there is uniform flow around particles is investigated.

1. FOR LOW magnetic Reynolds number R_m the system of equations of magnetohydrodynamics, which describes the motion of a homogeneous liquid in the natural magnetic field of constant electric currents flowing through it, supplied from an external source, has the form

$$\operatorname{div}\mathbf{u}=0,\quad \rho[\partial\mathbf{u}/\partial t+(\mathbf{u}\nabla)\mathbf{u}]=-\nabla p+\mu\Delta\mathbf{u}+c^{-1}\mathbf{j}\times\mathbf{H}+\rho\mathbf{g} \tag{1.1}$$

$$\operatorname{rot}\mathbf{H}=4\pi c^{-1}\mathbf{j},\quad \operatorname{div}\mathbf{H}=0,\quad \operatorname{rot}\mathbf{E}=0,\quad \mathbf{j}=\sigma\mathbf{E} \tag{1.2}$$

Here $\mathbf{u}$ is the velocity, ρ and p are the density and the pressure, $\mathbf{H}$ and $\mathbf{E}$ are the magnetic and electric fields, $\mathbf{g}$ is the acceleration due to gravity, μ and σ are the dynamic viscosity and the conductivity, and c is the velocity of light. In view of the smallness of R_m we can neglect the difference in the electric fields in frames of reference moving relative to one another. In writing Eqs (1.1) a fixed rectangular Cartesian system of coordinates x_1, x_2, x_3 is used. We will consider the case of a spatially non-uniform distribution of the current density j, so that the Lorentz forces have a vortex form [2]. Henceforth $\mathbf{H}(\mathbf{x})$, $\mathbf{E}(\mathbf{x})$, $\mathbf{u}(\mathbf{x}, t)$, $p(\mathbf{x}, t)$, $\mathbf{x}=(x_1, x_2, x_3)$ will mean the solution of the specific problem formulated for the system of equations (1.1) and (1.2).

Suppose that at a certain instant of time, drops of an immiscible conducting liquid are placed in the flow considered. Neglecting the hydrodynamic and electromagnetic interaction of the drops we will calculate the force acting on an individual drop. To do this we will use the approach employed in ordinary hydrodynamics when calculating the forces acting on a particle in a gradient flow at low Reynolds numbers, characterizing the motion of the liquid relative to the particle [3].

We will denote the trajectory of the centre of gravity O of the drop considered by $\mathbf{x}=\mathbf{X}(t)$

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and we will introduce a moving rectangular system of coordinates $\xi_i = x_i - X_i$ $(i = 1, 2, 3)$ with origin at the point O, which, together with the drop, performs translational motion with velocity $\mathbf{V} = d\mathbf{X}/dt$. In this case, the flow of the liquid, considered with respect to the non-inertial system of coordinates ξ_1, ξ_2, ξ_3 (the relative motion) is described by the equations

$$\begin{aligned} &\operatorname{div}\mathbf{u}_k = 0,\ \ \rho_k[\partial\mathbf{u}_k/\partial t + (\mathbf{u}_k\nabla)\mathbf{u}_k] = \\ &= -\nabla p_k + \mu_k\Delta\mathbf{u}_k + (4\pi)^{-1}\operatorname{rot}\mathbf{H}_k\times\mathbf{H}_k + \rho_k(\mathbf{g} - d\mathbf{V}/dt),\ \ k = 1,2 \end{aligned} \tag{1.3}$$

Quantities with the subscript 1 relate to the drop, and those with subscript 2 relate to the carrying liquid. The equations for the fields $\mathbf{H}_k$ and $\mathbf{E}_k$ are similar to (1.2). After eliminating $\mathbf{E}_1$ and $\mathbf{E}_2$ from these equations we obtain

$$\Delta\mathbf{H}_k = 0,\ \ \operatorname{div}\mathbf{H}_k = 0,\ \ k = 1,2 \tag{1.4}$$

Suppose Γ is the surface of the drop, and $\eta(\xi_1,\ \xi_2,\ \xi_3,\ t) = 0$ is its equation. The following kinematic, dynamic and electromagnetic conditions are satisfied on the surface Γ

$$\partial\eta/\partial t + |\nabla\eta|u_{1n} = 0,\ \ \mathbf{u}_1 = \mathbf{u}_2,\ \ \mathbf{p}_{n\tau}^{(2)} = \mathbf{p}_{n\tau}^{(1)},\ \ p_{nn}^{(2)} - p_{nn}^{(1)} = \alpha\operatorname{div}\mathbf{n} \tag{1.5}$$

$$\mathbf{H}_1 = \mathbf{H}_2,\ \ \sigma_1^{-1}(\operatorname{rot}\mathbf{H}_1)_\tau = \sigma_2^{-1}(\operatorname{rot}\mathbf{H}_2)_\tau \tag{1.6}$$

where $\mathbf{p}_n = p_{ij}\mathbf{э}_i n_j$, p_{ij} is the hydrodynamic part of the stress tensor, $\mathbf{э}_i$ are the vectors of the basis of the system of coordinates ξ_1, ξ_2, ξ_3, $\mathbf{n} = n_i\mathbf{э}_i$ is the unit vector of the outward normal to Γ, and α is the surface tension coefficient; the subscripts n and τ denote the components of the corresponding vectors normal and tangential to Γ. The following conditions are satisfied far from the drop

$$|\xi|/a \to \infty:\ \ \mathbf{u}_2 \to \mathbf{u} - \mathbf{V},\ \ \mathbf{H}_2 \to \mathbf{H} \tag{1.7}$$

where a is the characteristic size of the drop.

When there is no drop the equations of relative motion of the liquid have the form

$$\operatorname{div}\mathbf{U} = 0,\ \ \rho_2[\partial\mathbf{U}/\partial t + (\mathbf{U}\nabla)\mathbf{U}] = -\nabla P + \mu_2\Delta\mathbf{U} + (4\pi)^{-1}\operatorname{rot}\mathbf{H}\times\mathbf{H} + \rho_2(\mathbf{g} - d\mathbf{V}/dt) \tag{1.8}$$

where $\mathbf{U}(\xi,\ t) = \mathbf{u}(\mathbf{x},\ t) - \mathbf{V}(t)$. Suppose l is the characteristic scale of the distance over which the velocity $\mathbf{U}$ and the magnetic field $\mathbf{H}$ undergo considerable changes; then, naturally, $\varepsilon = a/l \ll 1$. We will assume that the characteristic relative velocity U_* is small, so that the Reynolds number R constructed with respect to U_* and with respect to the length a is small.

The magnetic fields and the distributions of the hydrodynamic parameters outside the drop will be sought in the form

$$\mathbf{H}_k = \mathbf{H} + \mathbf{h}_k,\ \ k = 1,2;\ \ \mathbf{u}_2 = \mathbf{U} + \mathbf{v},\ \ p_2 = P + q \tag{1.9}$$

The scale of the distance over which a considerable change in the perturbations of h_2, $\mathbf{v}$ and q occurs, is determined, in order of magnitude, by the dimensions of the drop. Substituting (1.9) into (1.3) and taking (1.8) into account, we obtain after neglecting quantities of the orders of $R, \varepsilon R$, $R_\upsilon = \rho_2\upsilon_* a/\mu_2$

$$\begin{aligned} &\operatorname{div}\mathbf{v} = 0,\ \ \rho_2\partial\mathbf{v}/\partial t = -\nabla q + \mu_2\Delta\mathbf{v} + \boldsymbol{\psi}_2 \\ &\boldsymbol{\psi}_2 = (4\pi)^{-1}(\operatorname{rot}\mathbf{H}_2\times\mathbf{h}_2 + \operatorname{rot}\mathbf{h}_2\times\mathbf{H}) \end{aligned} \tag{1.10}$$

Inside the drop in the Stokes approximation, we have

$$\operatorname{div}\mathbf{u}_1 = 0, \quad \rho_1\partial\mathbf{u}_1/\partial t = -\nabla p_1 + \mu_1\Delta\mathbf{u}_1 + \psi_1 + \rho_1(\mathbf{g} - d\mathbf{V}/dt)$$
$$\psi_1 = (4\pi)^{-1}\operatorname{rot}\mathbf{H}_1 \times \mathbf{H}_1 \tag{1.11}$$

The equations for the perturbations of the magnetic field produced by the drop retain their initial form (1.4)

$$\Delta\mathbf{h}_k = 0, \quad \operatorname{div}\mathbf{h}_k = 0, \quad k = 1,2 \tag{1.12}$$

It is assumed that the drop is spherical at the instant it is placed in the liquid. Later, a non-uniform distribution of the normal component p_{nn}^2 of the stress vector is formed on its surface when the drop moves, which changes the initial form of the drop. When there is no electromagnetic field, in the case of steady-state motion of the drop in a liquid at rest, for low Reynolds and Weber numbers, the surface-tension forces under the last dynamic condition (1.5) considerably exceed the non-uniformity of the distribution of the normal stresses, in view of which, the departure of the shape of the drop from a sphere is small [4]. When calculating the forces acting on the drop using the unsteady Stokes equations (a bibliography can be found in [5]) we will neglect the deformation of the spherical drop. In the case considered we will assume that the condition $We = \rho_2 U_*^2 a/\alpha \ll 1$ is satisfied, and we will ignore the deformation of the drop in view of its smallness.

We will introduce a spherical system of coordinates r, ϑ, φ with pole at the point O: $\xi_1 = r\sin\vartheta\cos\varphi$, $\xi_2 = r\sin\vartheta\sin\varphi$, $\xi_3 = r\sin\vartheta$. Neglecting the departure of the form of the drop from a sphere $r = a$, the boundary conditions (1.5) and (1.6), taking (1.9) into account, can be written in the form

$$u_{1n} = 0, \quad v_n = -U_n^0, \quad \mathbf{u}_{1\tau} - \mathbf{v}_\tau = \mathbf{U}_\tau^0 \tag{1.13}$$

$$\mu_1\partial\mathbf{u}_{1\tau}/\partial r - \mu_2\partial\mathbf{v}_\tau/\partial r = (\mu_1 - \mu_2)a^{-1}\mathbf{u}_{1\tau}, \quad \mathbf{U}^0 = \mathbf{U}(0,t)$$
$$\mathbf{h}_1 = \mathbf{h}_2, \quad \sigma_1^{-1}(\operatorname{rot}\mathbf{h}_1)_\tau - \sigma_2^{-1}(\operatorname{rot}\mathbf{h}_2)_\tau = (\sigma_2^{-1} - \sigma_1^{-1})(\operatorname{rot}\mathbf{H}|_{\mathbf{x}=\mathbf{X}})_\tau \tag{1.14}$$

On the right-hand sides of the second and third boundary conditions (1.13), and also in the second condition (1.14) we have omitted small quantities of the order of ε. Substituting (1.9) into (1.7) we obtain

$$r/a \to \infty: \quad \mathbf{v} \to 0, \quad \mathbf{h}_2 \to 0 \tag{1.15}$$

We will assume that at the initial instant of time, the liquids are at rest

$$t = 0: \quad \mathbf{u}_1 = 0, \quad \mathbf{v} = 0, \quad \mathbf{U} = 0, \quad \mathbf{V} = 0 \tag{1.16}$$

Hence, a calculation of the perturbations of the hydrodynamic and magnetic fields caused by the drop reduces to solving problem (1.10)–(1.16). Naturally, only bounded solutions with bounded derivatives have any physical meaning.

2. When calculating the force F acting on the drop, we need to take into account the action of the carrying liquid and of the electromagnetic field

$$\mathbf{F} = \mathbf{F}_l + \mathbf{F}_m; \quad \mathbf{F}_l = \int\limits_{r=a} \mathbf{p}_n^{(2)}d\sigma, \quad \mathbf{F}_m = \int\limits_{r=a} \mathbf{T}_n^{(2)}d\sigma, \quad \mathbf{T}_n = T_{ij}\mathbf{э}_i n_j \tag{2.1}$$

$$T_{ij} = (4\pi)^{-1}(H_iH_j - \tfrac{1}{2}H^2\delta_{ij}); \quad \delta_{ij} = 1 \text{ when } i = j, \quad \delta_{ij} = 0 \text{ when } i \neq j$$

In the approach used here $p_{ij}^{(2)}$ can be written in the same way as the representation of the hydrodynamic field (1.9)

$$p_{ij}^{(2)} = \Pi_{ij} + \pi_{ij};\quad \Pi_{ij} = -P\delta_{ij} + \mu_2\left(\frac{\partial U_i}{\partial \xi_j} + \frac{\partial U_j}{\partial \xi_i}\right),\quad \pi_{ij} = -q\delta_{ij} + \mu_2\left(\frac{\partial \upsilon_i}{\partial \xi_j} + \frac{\partial \upsilon_j}{\partial \xi_i}\right)$$

so that the first term in the expression for the force (2.1) can be converted, using Gauss' formula, to the form

$$\mathbf{F}_l = \mathbf{F}_l^0 + \mathbf{F}_l^1;\quad \mathbf{F}_l^0 = \int_{r\le a} (-\nabla P + \mu_2 \Delta \mathbf{U})d\tau,\quad \mathbf{F}_l^1 = \int_{r=a} \pi_n d\sigma,\quad \pi_n = \pi_{ij}\mathbf{э}_i n_j \tag{2.2}$$

We can similarly convert the second term

$$\mathbf{F}_m = \frac{1}{4\pi}\int_{r\le a}\left[(\mathbf{H}_1\nabla)\mathbf{H}_1 - \frac{1}{2}\nabla \mathbf{H}_1^2\right]d\tau = \frac{1}{4\pi}\int_{r\le a} \operatorname{rot}\mathbf{H}_1 \times \mathbf{H}_1 d\tau = \mathbf{F}_m^0 + \mathbf{F}_m^1$$

$$\mathbf{F}_m^0 = \frac{1}{4\pi}\int_{r\le a} \operatorname{rot}\mathbf{H}\times\mathbf{H}d\tau,\quad \mathbf{F}_m^1 = \frac{1}{4\pi}\int_{r\le a} (\operatorname{rot}\mathbf{h}_1 \times \mathbf{H}_1 + \operatorname{rot}\mathbf{H}\times\mathbf{h}_1)d\tau$$

As a result, when the equation of relative motion of the liquid (1.8) is taken into account the formula for the force (2.1) can be written in the form

$$\mathbf{F} = \mathbf{F}^0 + \mathbf{F}_l^1 + \mathbf{F}_m^1;\quad \mathbf{F}^0 = \mathbf{F}_l^0 + \mathbf{F}_m^0 = \rho_2 \int_{r\le a}\left[\left(\frac{d\mathbf{U}}{dt} + \frac{d\mathbf{V}}{dt}\right) - \mathbf{g}\right]d\tau \tag{2.3}$$

The expression in parentheses is the acceleration $d\mathbf{u}/dt$ of a particle of the liquid with respect to the fixed system of coordinates x_1, x_2, x_3. Neglecting the change in $d\mathbf{u}/dt$ in the region of integration, we obtain

$$\mathbf{F}^0 = \frac{4\pi a^3}{3}\rho_2\left(\left.\frac{d\mathbf{u}}{dt}\right|_{\mathbf{x}=\mathbf{X}(t)} - \mathbf{g}\right)$$

This expression is identical in form to that obtained in [3], but here the acceleration $d\mathbf{u}/dt$ depends on the distribution of the Lorentz forces in the liquid.

To calculate the force $\mathbf{F}_m^1$, produced by the perturbation of the electromagnetic field caused by the drop, we need to obtain a solution of problem (1.12), (1.14) and (1.15). When constructing the solution we will use [6] vector spherical harmonics $\mathbf{P}_{mn}(\vartheta,\ \varphi)$, $\mathbf{B}_{mn}(\vartheta,\ \varphi)$, $\mathbf{C}_{mn}(\vartheta,\ \varphi)$

$$\mathbf{P}_{mn} = Y_{mn}\mathbf{a}_r,\quad Y_{mn} = e^{im\varphi}P_n^m(\cos\vartheta);\quad m = 0,1,\ldots,n;\quad n = 0,1,2,\ldots$$

$$\mathbf{B}_{mn} = \frac{r}{\sqrt{n(n+1)}}\nabla Y_{mn},\quad \mathbf{C}_{mn} = \frac{1}{\sqrt{n(n+1)}}\operatorname{rot}[rY_{mn}\mathbf{a}_r];\quad \begin{matrix} m = 0,1,\ldots,n; \\ n = 1,2,\ldots \end{matrix}$$

with angular functions $\mathbf{D}_{mn}^\alpha(\vartheta,\ \varphi)$ $(\alpha = 1, \ldots, 6)$, which form a complete orthogonal system on the sphere

$$\mathbf{D}_{mn}^1 = \sqrt{n(n+1)}\mathbf{C}_{mn}^e,\quad \mathbf{D}_{mn}^3 = \sqrt{n+1}[\sqrt{n+2}\mathbf{B}_{m,n+1}^e + \sqrt{n+1}\mathbf{P}_{m,n+1}^e]$$

$$\mathbf{D}_{mn}^5 = \sqrt{n}[\sqrt{n-1}\mathbf{B}_{m,n-1}^e - \sqrt{n}\mathbf{P}_{m,n-1}^e]$$

Here $\mathbf{a}_r$ is the unit vector along the coordinate line r, the superscript e denotes the real parts of the corresponding expressions, and the superscript 0 denotes the imaginary parts. The expressions $\mathbf{D}_{mn}^2$, $\mathbf{D}_{mn}^4$, $\mathbf{D}_{mn}^6$ are obtained from $\mathbf{D}_{mn}^1$, $\mathbf{D}_{mn}^3$, $\mathbf{D}_{mn}^5$ by replacing e by 0.

The unperturbed magnetic field, which occurs in the boundary conditions (1.14), has the following representation in the neighbourhood of the point O, apart from quantities of small orders in r/l

$$\mathbf{H}=\mathbf{H}^0+r\left[\mathbf{A}+\frac{\varepsilon_{11}^0-\varepsilon_{22}^0}{12}\mathbf{D}_{21}^3+\frac{\varepsilon_{33}^0}{2}\mathbf{D}_{01}^3+\frac{\varepsilon_{12}^0}{6}\mathbf{D}_{21}^4+\frac{\varepsilon_{13}^0}{3}\mathbf{D}_{11}^3+\frac{\varepsilon_{23}^0}{3}\mathbf{D}_{11}^4\right]$$

$$\mathbf{A}=2\pi c^{-1}(j_1^0\mathbf{D}_{11}^1+j_2^0\mathbf{D}_{11}^2+j_3^0\mathbf{D}_{01}^1) \tag{2.4}$$

$$\varepsilon_{ij}^0=\tfrac{1}{2}(\partial H_i/\partial x_j+\partial H_j/\partial x_i)_{\mathbf{x}=\mathbf{X}},\quad \mathbf{H}^0=\mathbf{H}(\mathbf{X}),\quad \mathbf{j}^0=\mathbf{j}(\mathbf{X})$$

Finding the solution of problem (1.12), (1.14) and (1.15) in the form of the sum of solenoidal partial solutions of Laplace's vector equation [6], which is expanded in the same system of vector spherical harmonics as the expression in square brackets on the right-hand side of (2.4), we obtain

$$\mathbf{h}_1=(\kappa-1)r\mathbf{A},\quad \mathbf{h}_2=(\kappa-1)a^3r^{-2}\mathbf{A},\quad \kappa=3\sigma_1/(\sigma_1+2\sigma_2) \tag{2.5}$$

Using expressions (2.4) and (2.5) we can calculate $\mathbf{F}_m^1$, and also the density of the Lorentz forces ψ_1 inside the drop and the perturbation of the Lorentz forces ψ_2 in the carrying liquid produced by the drop. Confining ourselves to the principal terms of the expansions with respect to the small parameter r/l, we have

$$\mathbf{F}_m^1=\tfrac{4}{3}\pi a^3(\kappa-1)\mathbf{f}^0,\quad \mathbf{f}^0=c^{-1}\mathbf{j}^0\times\mathbf{H}^0$$

$$\psi_1=\kappa\mathbf{f}^0,\quad \psi_2=2\pi(\kappa-1)c^{-1}(a/r)^3\mathbf{H}^0\times(j_1^0\mathbf{D}_{12}^5+j_2^0\mathbf{D}_{12}^6+j_3^0\mathbf{D}_{02}^5) \tag{2.6}$$

In the approximation considered $\operatorname{rot}\psi_1=0$, whereas $\operatorname{rot}\psi_2\neq 0$.

3. The method described in [1] for solving boundary-value problems for the steady Stokes equations containing vortex volume forces is difficult to generalize to the unsteady case. Using this method, the velocity field is constructed in the form of an expansion in angular functions $\mathbf{D}_{mn}^\alpha$, while the pressure field is constructed in the form of an expansion in spherical functions Y_{mn}^e, Y_{mn}^0. In the Stokes approximation, to calculate the force acting on the drop in a flow of a viscous incompressible liquid, only the terms of the expansion containing $\mathbf{D}_{00}^3$, $\mathbf{D}_{10}^3$, $\mathbf{D}_{10}^4$, Y_{01}, Y_{11}^e, Y_{11}^0 make a non-zero contribution [1]. In the expansion of the velocity field, to satisfy the boundary conditions on the surface of the drop, we must also take into account terms containing the angular functions $\mathbf{D}_{02}^5$, $\mathbf{D}_{12}^5$, $\mathbf{D}_{12}^6$, which are expressed in terms of the same vector spherical harmonics $\mathbf{P}_{01}$, $\mathbf{B}_{01}$, $\mathbf{P}_{11}^e$, $\mathbf{B}_{11}^e$, $\mathbf{P}_{11}^0$, $\mathbf{B}_{11}^0$ as $\mathbf{D}_{00}^3$, $\mathbf{D}_{10}^3$, $\mathbf{D}_{10}^4$. In the case considered of the unsteady Stokes equations, the parts v_k', p_k' ($k=1$, 2) of the hydrodynamic fields ($\mathbf{u}_i$, p_1), ($\mathbf{v}$, q), sufficient, using (2.2), to calculate the force

$$\mathbf{F}_l^1=\int_{r=a}\pi_n' d\sigma;\quad \pi_n'=-p_2'\mathbf{a}_r+\mu_2\left[\frac{\partial v_2'}{\partial r}-\frac{v_2'}{r}+\frac{1}{r}\nabla(rv_{2r}')\right] \tag{3.1}$$

must be sought in the form

$$\mathbf{v}_k'=\operatorname{rot}\operatorname{rot}\mathbf{w}_k,\quad p_k'=(\mu_k\Delta-\rho_k\partial/\partial t)\operatorname{div}\mathbf{w}_k \tag{3.2}$$

$$\mathbf{w}_k=x_{k1}(r,t)\mathbf{D}_{10}^3+x_{k2}(r,t)\mathbf{D}_{10}^4+x_{k3}(r,t)\mathbf{D}_{00}^3+$$
$$+y_{k1}(r,t)\mathbf{D}_{12}^5+y_{k2}(r,t)\mathbf{D}_{12}^6+y_{k3}(r,t)\mathbf{D}_{02}^5 \tag{3.3}$$

By substituting (3.3) into (3.2) we obtain the following form of the required part of the hydrodynamic fields

$$\mathbf{v}_k' = -\tfrac{2}{3}[T(x_{k1},y_{k1})\mathbf{D}_{10}^3 + T(x_{k2},y_{k2})\mathbf{D}_{10}^4 + T(x_{k3},y_{k3})\mathbf{D}_{00}^3] -$$
$$-\tfrac{1}{3}[S(x_{k1},y_{k1})\mathbf{D}_{12}^5 + S(x_{k2},y_{k2})\mathbf{D}_{12}^6 + S(x_{k3},y_{k3})\mathbf{D}_{02}^5]$$
$$p_k' = \left(\mu_k L_1 - \rho_k \frac{\partial}{\partial t}\right)\left\{Y_{11}^e\left[\frac{\partial x_{k1}}{\partial r} - 2\left(\frac{\partial y_{k1}}{\partial r} + \frac{3y_{k1}}{r}\right)\right] + Y_{11}^0\left[\frac{\partial x_{k2}}{\partial r} -\right.\right.$$
$$\left.\left.-2\left(\frac{\partial y_{k2}}{\partial r} + \frac{3y_{k2}}{r}\right)\right] + Y_{01}\left[\frac{\partial x_{k3}}{\partial r} - 2\left(\frac{\partial y_{k3}}{\partial r} + \frac{3y_{k3}}{r}\right)\right]\right\}$$
$$T(x_{ki},y_{ki}) = L_0(x_{ki}+y_{ki}) + \frac{3}{r}\left(\frac{\partial y_{ki}}{\partial r} + \frac{y_{ki}}{r}\right)$$
$$S(x_{ki},y_{ki}) = L_0(x_{ki}+y_{ki}) - \frac{3}{r}\left(\frac{\partial x_{ki}}{\partial r} + \frac{2y_{ki}}{r}\right), \quad i=1,2,3$$
$$L_n = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{n(n+1)}{r^2}, \quad n=0,1,2$$

The equations, boundary conditions and initial conditions for $\mathbf{v}_k'$, p_k' are naturally identical with (1.10), (1.11), (1.13), (1.15) and (1.16). Substituting the representations (3.2) and (3.3) into these expressions and expanding $\psi_1 + \rho_1(\mathbf{g} - d\mathbf{V}/dt)$, ψ_2 in series in the complete system of angular functions, we arrive at the following problems

$$L_0\left[\left(\mu_1 L_0 - \rho_1\frac{\partial}{\partial t}\right)x_{1i}\right] = a_{1i}, \quad L_2\left[\left(\mu_1 L_2 - \rho_1\frac{\partial}{\partial t}\right)y_{1i}\right] = 0 \tag{3.4}$$

$$L_0\left[\left(\mu_2 L_0 - \rho_2\frac{\partial}{\partial t}\right)x_{2i}\right] = 0, \quad L_2\left[\left(\mu_2 L_2 - \rho_2\frac{\partial}{\partial t}\right)y_{2i}\right] = \frac{a_{2i}}{r^3} \tag{3.5}$$

$$r=a:\ T(x_{1i},y_{1i}) - S(x_{1i},y_{1i}) = 0, \quad T(x_{2i},y_{2i}) - S(x_{2i},y_{2i}) = \frac{3}{2}U_i^0 \tag{3.6}$$
$$2[T(x_{2i},y_{2i}) - T(x_{1i},y_{1i})] + S(x_{2i},y_{2i}) - S(x_{1i},y_{1i}) = 3U_i^0$$
$$\mu_2\frac{\partial}{\partial r}[2T(x_{2i},y_{2i}) + S(x_{2i},y_{2i})] - \mu_1\frac{\partial}{\partial r}[2T(x_{1i},y_{1i}) + S(x_{1i},y_{1i})] =$$
$$= \frac{\mu_2-\mu_1}{a}[2T(x_{1i},y_{1i}) + S(x_{1i},y_{1i})]$$
$$r/a\to\infty:\ T(x_{2i},y_{2i})\to 0, \quad S(x_{2i},y_{2i})\to 0$$
$$t=0:\ x_{ki}=0, \quad y_{ki}=0$$

The right-hand sides of Eqs (3.4) and (3.5) are the coefficients in the expansions of $\psi_1 + \rho_1(\mathbf{g} - d\mathbf{V}/dt)$, ψ_2 in front of the same angular functions with which the required functions x_{ki} and y_{ki} occur in representation (3.3). Reverting to (2.6) we obtain

$$a_{1i} = \kappa f_i^0 + \rho_1(g_i - dV_i/dt), \quad a_{2i} = \tfrac{1}{4}a^3(\kappa-1)f_i^0, \quad i=1,2,3$$

Using a Laplace transformation with respect to time, we obtain solutions of the operator problems corresponding to Eqs (3.4) and (3.5) and boundary conditions (3.6). The transform $L(\pi_n')$ at the surface of the drop (with $r=a$), calculated using these solutions, has the form

$$L[\pi_n'(a,\vartheta,\varphi,t)] = \frac{\mu_2}{2a}\{[12 + \theta s + 3\delta(\theta s)(\sqrt{\theta s} - 3)]L(\mathbf{U}_n^0) -$$
$$-3[2 - \delta(\theta s)(\sqrt{\theta s} + 3)]L(\mathbf{U}_\tau^0)\} - \frac{a(\kappa-1)}{4s}\mathbf{f}_n^0; \quad \theta = \frac{\rho_2 a^2}{\mu_2} \tag{3.7}$$

$$\delta(\theta s)=\frac{\mu_1\beta(\lambda)+2\mu_2\gamma(\lambda)}{\mu_1\beta(\lambda)+\mu_2\gamma(\lambda)(\sqrt{\theta s}+3)},\quad \lambda=\frac{\sqrt{\theta s}}{B},\quad B=\frac{\rho_2\mu_1}{\rho_1\mu_2}$$

$$\beta(\lambda)=\lambda\operatorname{ch}\lambda(6+\lambda^2)-3\operatorname{sh}\lambda(2+\lambda^2),\quad \gamma(\lambda)=\operatorname{sh}\lambda(3+\lambda^2)-3\lambda\operatorname{ch}\lambda$$

where s is the parameter of the Laplace transformation. Integrating (3.7) over the surface of the drop and carrying out an inverse Laplace transformation, we obtain

$$\mathbf{F}_l^1=\frac{2\pi a\mu_2(3\mu_1+2\mu_2)}{\mu_1+\mu_2}\{\mathbf{u}[\mathbf{X}(t),t]-\mathbf{V}(t)\}+\frac{2\pi a^3}{3}\rho_2\frac{d}{dt}\{\mathbf{u}[\mathbf{X}(t),t]-\mathbf{V}(t)\}+$$
$$+6\pi a\mu_2\int_0^t I_1\left(\frac{t-\tau}{\theta}\right)\frac{d}{d\tau}\{\mathbf{u}[\mathbf{X}(\tau),\tau]-\mathbf{V}(\tau)\}d\tau-\frac{\pi a^3}{3}(\kappa-1)\mathbf{f}^0 \tag{3.8}$$

Here $I_1(t/\theta)$ is the original of the transform

$$K_1(\theta s)=\frac{1}{\theta s}\left[\delta(\theta s)(\sqrt{\theta s}+1)-\frac{3\mu_1+2\mu_2}{3(\mu_1+\mu_2)}\right]$$

Graphs of the function $\pi^{-1}I_1(t/\theta)$, obtained numerically for different values of the parameters μ_1/μ_2, B are given in [5]. In the approximation considered, as in the case of solid particles [2], the readjustment of the flow in the region of the drop due to the electromagnetic field has no effect on the drag of the drop. Passing to the limit in (3.8) as $\mu_1/\mu_2\to 0$, $\mu_2=\text{const}$ we can obtain an expression for the force in the case of a solid particle.

Using (2.6) and (3.8) we obtain the overall force of electromagnetic origin, acting on the drop

$$\mathbf{F}_s=\pi a^3(\kappa-1)\mathbf{f}^0 \tag{3.9}$$

When the current distribution in the carrying liquid remains unchanged (when there is no drop), $\mathbf{F}_s$ reverses its direction on changing from case $\sigma_1>\sigma_2$ to case $\sigma_1<\sigma_2$.

We will consider briefly the problem of the electromagnetic force acting on a non-conducting ($\sigma_1=0$) drop, around which an electric current flows in a conducting liquid. The magnetic field inside the drop is quasi-steady, so that, with sufficient accuracy, we can use the equations

$$\operatorname{rot}\mathbf{H}_1=0,\quad \operatorname{div}\mathbf{H}_1=0 \tag{3.10}$$

whereas the magnetic field $\mathbf{H}_2$ in the carrying liquid is described by Eqs (1.4). Unlike the case when $\sigma_1\neq 0$ only the first condition of (1.6) is imposed at the surface of the non-conducting drop—the requirement that the magnetic field should be continuous. Here, the continuity of the tangential component of the magnetic field at the drop surface, together with the first equation of (3.10), automatically ensures that the normal component of the current density vanishes when $r=a$.

After calculating the magnetic fields, following the procedure described above as it applies to a non-conducting drop, we obtain $\mathbf{F}_s=-\pi a^3\mathbf{f}^0$, i.e. Eq. (3.9) can be used both for a conducting drop and for a non-conducting drop.

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